T-duality for open strings with respect to non-abelian isometries[1]

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Abstract

We gauge the non-abelian isometries of a sigma model with boundaries. Forcing the field strength of the gauge fields to vanish renders the gauged model equivalent to the ungauged one provided that boundary conditions are taken into account properly. Integrating out the gauge fields gives the T-dual model. We observe that T-duality interchanges Neumann (or mixed) boundary conditions with Dirichlet boundary conditions.

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1   Introduction

Some time ago it has been observed that T-duality in toroidally compactified open strings interchanges Neumann with Dirichlet boundary conditions [1]. In the recent past there has been renewed interest in this subject since it provides extended objects (D-branes) which are important in the context of string dualities, (for an excellent review see [2]). Therefore it is of special interest to generalize the T-duality transformation for open strings living in a more general background.

The first step in this direction has been done in [3] where backgrounds possessing a Poisson-Lie symmetry have been considered. In [4] and independently in [5] T-duality was carried out in general backgrounds with Abelian isometries. The present talk is addressed to the generalization for backgrounds with non-Abelian isometries, work published in [6] will be reported.

Some papers dealing with T-duality corresponding to non-Abelian isometries are listed in [7, 8]. Here we will follow the initial work of [7], and we restrict ourself to semi simple isometry groups.

In the second section some preliminary consideration discussing boundary conditions in a setting suitable for general backgrounds will be presented. How to gauge a non-Abelian isometry in a sigma model with boundary will be shown in the third section. Finally, the T-duality will be performed in the fourth section, followed by concluding remarks in a fifth section.

2   Preliminary consideration

In this section we discuss how boundary conditions transform under T-duality. For simplicity we will consider just one free boson on a world sheet with boundary, the generalization to less trivial cases will be straightforward,

$$S = \int_\Sigma d^2z \, \partial_a X \partial^a X. \tag{1}$$

This action is invariant under constant shifts in X. The first step in deriving the T-dual model is to gauge this global symmetry, (for a review on T-duality see [9]). In order to obtain an action invariant under the local transformation

$$X \to X + f(z) \tag{2}$$

we have to introduce two dimensional gauge fields Ω_a transforming according to

$$\Omega_a \to \Omega_a - \partial_a f. \tag{3}$$

and to replace partial derivatives by covariant ones,

$$\partial_a X \to D_a X = \partial_a X + \Omega_a. \tag{4}$$

A gauge invariant action is given by, (we neglect global issues since these can not be discussed in the non-Abelian case [8]),

$$S_{gauged} = \int_\Sigma d^2 z \, (D_a X D^a X + \lambda F) + \oint_{\partial\Sigma} ds \, ct^a \Omega_a, \tag{5}$$

where F is the field strength corresponding to the isometry gauge field Ω,

$$F = \epsilon^{ab} \partial_a \Omega_b, \tag{6}$$

λ is a Lagrange multiplier forcing the field strength F to vanish, t is the tangent vector on the boundary $\partial\Sigma$ and c is an arbitrary constant. Now, we consider the partition function

$$Z = \int \mathcal{D}\Omega \mathcal{D} X \mathcal{D}\lambda e^{-S_{gauged}}. \tag{7}$$

Integrating out the Lagrange multiplier λ forces the field strength F to vanish which in turn constrains the gauge field Ω to be "pure gauge", i.e. $\Omega_a = \partial_a \rho$. Upon the redefinition

$$X \to X + \rho \tag{8}$$

the original model is obtained back. However, since we are considering a model with boundary we have to be careful that the shift (8) does not change the boundary condition. Therefore, let us specify the boundary condition in the original model by

$$b^a \partial_a X_{|\partial\Sigma} = 0. \tag{9}$$

(If the vector b is normal to the boundary these are Neumann conditions whereas b being tangent to the boundary gives - up to a constant - Dirichlet conditions.) The (local) equivalence to the original model can be ensured by adding a second Lagrange multiplier term $\oint ds \, \kappa b^a \Omega_a$ on the boundary. Now, the gauged action (after partial integration) reads

$$\begin{aligned} S_{gauged} = & \int d^2 z \left(\partial_a X \partial^a X + \Omega_a \Omega^a + 2\Omega_a \partial^a X - \epsilon^{ab} \partial_a \lambda \Omega_b \right) \\ & + \oint ds \, (ct_a + \kappa b_a + t_a \lambda) \, \Omega^a. \end{aligned} \tag{10}$$

The T-dual model is obtained by integrating out the gauge fields Ω. The action (10) is written such that it contains no derivatives of Ω. Hence, the Ω integral is ultra local and integrations over gauge fields with arguments in the bulk and over gauge fields with arguments on the boundary factorize [5],

$$\int \mathcal{D}\Omega_{\Sigma \cup \partial\Sigma} (\ldots) = \int \mathcal{D}\Omega_\Sigma (\ldots) \times \int \mathcal{D}\Omega_{\partial\Sigma} (\ldots). \tag{11}$$

2

Integrating out gauge fields where the argument is in the bulk of the world sheet gives the dual action, $\tilde{S} = \int d^2z \, \partial_a \lambda \partial^a \lambda$. Since on the boundary Ω appears linearly that integration gives a two dimensional delta function

$$\delta^{(2)} \left(c t_a + \kappa b_a + \lambda t_a \right), \tag{12}$$

specifying the boundary conditions of the dual model as we will see now. There are essentially two different cases to be considered,

(1) $b \nparallel t$

(2) $b \parallel t$.

In case **(1)** the arguments of the two-dimensional delta function can vanish simultaneously only if $\kappa = 0$, and we are left with the dual boundary condition

$$\lambda_{|\partial\Sigma} = -c. \tag{13}$$

In case **(2)** the two-dimensional delta function just identifies the second Lagrange multiplier κ with the negative of the first one λ on the boundary, (in this case we can put the constant c to zero since the tangent component of the gauge field is zero),

$$\kappa = -\lambda_{|\partial\Sigma}. \tag{14}$$

So, the final picture we see is described as follows:

(1) We start with Neumann or mixed boundary conditions. This can be thought of as obtained from a general open string sigma model by requiring the variation of the action to vanish for free varying ends of the string ($\delta X_{|\partial\Sigma} = any$), (the target space metric is required to be non degenerate). In the dual model we obtain Dirichlet boundary conditions (13) enforced by a delta function, i.e. $\delta\lambda_{|\partial\Sigma} = 0$.

(2) We start with Dirichlet boundary conditions (up to a constant) enforced by a delta function, i.e. the variations of the end of the string are frozen to vanish. In the dual model no boundary conditions are specified. So, a natural choice is to pick those boundary conditions obtained from an action principle with free varying ends.

We have seen in this simple setting that an open string with free varying ends is T-dual to an open string whose ends are fixed on a surface in the target space, the D-brane.

3 Gauging the non-Abelian isometries

The general sigma model describing the action of an open string in a non trivial background is given by, (since any consistent open string theory includes closed strings there are also a metric and an antisymmetric tensor field),

$$
\begin{aligned}
S = \; & -\tfrac{1}{2} \int_\Sigma d^2\sigma \left(G_{mn} \partial_a X^m \partial^a X^n + B_{mn} \epsilon^{ab} \partial_a X^m \partial_b X^n \right) \\
& - \int_{\partial\Sigma} ds A_m \partial_s X^m + \tfrac{1}{4\pi} \int_\Sigma d^2\sigma \sqrt{\gamma} R^{(2)} \Phi - \tfrac{1}{2\pi} \int_{\partial\Sigma} ds k \Phi \,,
\end{aligned}
\tag{15}
$$

where Φ is the dilaton coupling to the Gauss-Bonnet density of the world sheet, i.e. to the scalar curvature R in the bulk and to the geodesic curvature k on the boundary. We assume that there are isometries, i.e. a set of Killing vectors $\mathcal{L}_{\xi_I} G_{mn} = 0$ satisfying a semi simple Lie-algebra, $[\xi_I, \xi_J] = f_{IJ}{}^K \xi_K$. Now we are going to specify the conditions under which the sigma model (15) is invariant with respect to the global transformations

$$
\delta X^m = \epsilon^I \xi_I^m, \tag{16}
$$

with ϵ^I being the constant transformation parameters. The first condition is that the Lie-derivative of the dilaton Φ has to vanish, and from now on we are excluding the dilaton from the discussion and will return to it in the conclusion when mentioning open problems. The Lie-derivatives of the antisymmetric tensor B and the $U(1)$-gauge field A have to vanish up to gauge transformations, i.e.

$$
\begin{aligned}
\mathcal{L}_{\xi_I} B_{mn} = \; & \partial_m \omega_{In} - \partial_n \omega_{Im}, \\
\mathcal{L}_{\xi_I} A_m = \; & \partial_m \phi_I + \omega_{Im},
\end{aligned}
\tag{17}
$$

where the ω and ϕ are arbitrary functions. The rhs of (17) is invariant under the redefinitions, $(\omega_I = \omega_{Im} dX^m)$,

$$
\begin{aligned}
\omega_I' = \; & \omega_I + dh_I, \\
\phi_I' = \; & \phi_I - h_I + k_I,
\end{aligned}
\tag{18}
$$

where the h_I are arbitrary functions and the k_I are constants. By evaluating the Lie-derivatives with respect to a commutator of two Killing vectors in two different ways we derive the following consistency conditions,

$$
\begin{aligned}
\mathcal{L}_{\xi_I} \omega_J - \mathcal{L}_{\xi_J} \omega_I = \; & f_{IJ}{}^K \omega_K + d\rho_{IJ}, \\
\mathcal{L}_{\xi_I} \phi_J - \mathcal{L}_{\xi_J} \phi_I = \; & f_{IJ}{}^K \phi_K - \rho_{IJ},
\end{aligned}
\tag{19}
$$

with ρ being some function. From [10, 11] we know that in the closed string case an anomaly free gauging is possible only if $d\rho_{IJ}$ vanishes. So, we have to remove ρ by employing the symmetry transformations (18). This can be done provided $d\rho$ satisfies certain integrability

conditions [10]. We assume that this is the case and set $d\rho_{IJ}$ to zero, i.e. $\rho_{IJ} = k_{IJ}$ to a constant. The possible symmetry transformations we are left with are constant shifts in ϕ_I under which the k_{IJ} transform. For simplicity we take the antisymmetric tensor B to be zero from now on.

Now we are going to gauge the model, i.e. ϵ in (16) depends on the position on the world sheet. We introduce isometry gauge fields Ω transforming as a connection

$$\delta\Omega_a^J = \partial_a\epsilon^I + f_{KJ}{}^I\Omega_a^K\epsilon^J. \tag{20}$$

Since we have put the antisymmetric tensor to zero the bulk part is gauged just by replacing partial derivatives with covariant ones,

$$D_aX^m = \partial_aX^m - \xi_I^m\Omega_a^I. \tag{21}$$

In order to gauge the boundary part we use Noether's method, i.e. we add the contribution

$$S^{(1)} = \int ds\, C_I\Omega_s^I, \tag{22}$$

where the index s denotes the tangent component. Requiring gauge invariance leads to two equations for C_I, *viz.*

$$\begin{aligned} C_I &= \quad -A_m\xi_I^m + \phi_I + \lambda_I, \\ \mathcal{L}_{\xi_I}C_J &= \quad -f_{JI}{}^K C_K, \end{aligned} \tag{23}$$

with λ_I constant. The two equations in (23) are compatible provided that

$$k_{IJ} - f_{IJ}{}^K\lambda_K = 0 \tag{24}$$

implying an integrability condition on the constants k_{IJ},

$$f_{IJ}{}^K k_{KL} + cycl.\ permutations = 0. \tag{25}$$

4 Non-Abelian T-duality

It is useful to split the coordinates into those which transform non trivially under the isometry gauge group carrying a Latin index and spectators carrying a Greek index. Using light cone coordinates on the world sheet, (for conventions cf [6]), the gauged action reads

$$\begin{aligned} S\left(X,\Omega\right) = \quad &\int_\Sigma d^2z\left\{G_{\alpha\beta}\partial X^\alpha\bar\partial X^\beta + G_{\alpha m}\left(\partial X^\alpha\bar DX^m + \bar\partial X^\alpha DX^m\right)\right. \\ &\left. + G_{mn}DX^m\bar DX^n + \Lambda_I G^I\right\} \\ &- \int_{\partial\Sigma} ds\left(A_\alpha\partial_sX^\alpha + A_m\partial_sX^m - C_I\Omega_s^I\right), \end{aligned} \tag{26}$$

5

where $G = \partial\bar{\Omega} - \bar{\partial}\Omega + [\Omega, \bar{\Omega}]$ is the field strength of the isometry gauge fields and the C_I are determined according to the previous section. The gauged action (26) is gauge invariant provided that the Lagrange multiplier Λ transforms in the adjoint, $\delta\Lambda_I = -f_{IJ}{}^K\Lambda_K\epsilon^J$. Here, we start with free varying ends of the string, i.e. Neumann or mixed boundary conditions. As we have seen in section 2 a (in principle necessary) second boundary-Lagrange multiplier drops out finally, and therefore we drop it already here. The first Lagrange multiplier forces the gauge field excitations to be pure gauge (locally) and after absorbing the gauge parameters in a field redefinition one obtains the original model back. The T-dual is constructed by integrating out the gauge fields. Again, we rewrite the action such that there are no derivatives of the isometry-gauge fields and employ the ultra locality to factorize the integral into a product of an integral with field arguments in the bulk and an integral with field arguments on the boundary. The bulk integral will provide the dual action coinciding with the closed string result [7]. The boundary integral specifies Dirichlet conditions on the dual coordinates enforced by a delta function,

$$\Lambda_I + C_{I\,|\partial\Sigma} = 0 \tag{27}$$

being covariant with respect to the isometry transformations. In addition one has to fix the gauge by fixing the X^m and in general also some of the Λ's. This concludes the construction of the T-dual model.

5   Conclusions

We have gauged the non-Abelian isometries in a general sigma model with boundary and constructed the T-dual model by integrating out the gauge fields. We observed that in the open string case T-duality interchanges Neumann or mixed boundary conditions (corresponding to free varying ends) with Dirichlet conditions enforced by a delta function, i.e. the variations of the ends of the string are fixed in the dual model.

Let us finish this talk with some open problems. First of all there are global issues which are hard to address also in the closed string case. In addition one should mention the dilaton. In a general sigma model with boundary there could be in principle two dilatons, one coupling to the curvature in the bulk and one coupling to the geodesic curvature on the boundary. A string interpretation requires only one dilaton coupling to the Gauss-Bonnet density, (that this is consistent with renormalizability has been shown in [12]). Now, a natural guess for the T-dual model would be that the dilaton shift is the same as in the closed string case [7, 13], and that in the dual model the dilaton couples just to the Gauss-Bonnet density as well. A proof of that guess is missing so far.

After this talk has been given ref. [14] appeared where results differing from ours have been obtained in a canonical transformation framework. The reason is that in the canonical transformation scheme one takes frequently variational derivatives and in ref. [14] periodic boundary conditions are imposed on variations. The canonical transformation and the method presented here are actually in agreement as discussed in [15].

Acknowledgments

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